DE89-007879

A NEURAL NETWORK - BASED OPTIMIZATION ALGORITHM FOR THE WEAPON-TARGET ASSIGNMENT PROBLEM

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NTIS

U02182

Accession Number: 2182

Publication Date: Feb 01, 1989

Title: Neural Network-based Optimization Algorithm for the Weapon-Target Assignment Problem

Personal Author: Wacholder, E.

Corporate Author Or Publisher: Oak Ridge National Laboratory, Oak Ridge, TN 38731 Report Number: ORNL/TM-11025

Descriptors, Keywords: Interceptor BMD BM C3 Algorithm Target Neural Network Assignment Weapon Simulation

Pages: 038

Cataloged Date: Jun 12, 1990

Contract Number: DE-AC05-84OR21400

Document Type: HC

Number of Copies In Library: 000001

Original Source Number: DE89-007879

Record ID: 21096

Source of Document: NTIS

DE89007879
ORNL/TM-11025

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E. Wacholder

OPERATED BY
MARTIN MARIETTA ENERGY SYSTEMS, INC.
FOR THE UNITED STATES
DEPARTMENT OF ENERGY

Printed in the United States of America. Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road, Springfield, Virginia 22161
NTIS price codes—Printed Copy: A03; Microfiche A01

ORNL/TM-11025

Engineering Physics and Mathematics Division

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Research was sponsored by the Office of Military Applications of the U.S. Department of Energy and by the U.S. Air Force Wright Aeronautical Laboratories, under Interagency Agreement DOE-40-1579-85

February 1989

Prepared by the
OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee 37831
operated by
MARTIN MARIETTA ENERGY SYSTEMS, INC.
for the
U.S. DEPARTMENT OF ENERGY
under contract DE–AC05–84OR21400

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ACKNOWLEDGMENTS

We thank Dr. J. Barhen, JPL; Professor B. Gavish, Vanderbilt University; and Dr. J. Han, ORNL, for helpful discussions and constructive comments. Special thanks are due to Dr. C. R. Weisbin, ORNL, for continuous encouragement and support, and to R. D. Lawson for expert preparation of this report.

ABSTRACT

A neural network-based algorithm was developed for the Weapon-Target Assignment Problem (WTAP) in Ballistic Missile Defense (BMD). An optimal assignment policy is one which allocates targets to weapon platforms such that the total expected leakage value of targets surviving the defense is minimized. This involves the minimization of a non-linear objective function subject to inequality constraints specifying the maximum number of interceptors available to each platform and the maximum number of interceptors allowed to be fired at each target as imposed by the Battle Management/Command Control and Communications (BM/C^3) system. The algorithm consists of solving a system of ODEs trajectories and variables. Simulations of the algorithm on PC and VAX computers were carried out using a simple numerical scheme. In all the battle instances tested, the algorithm has proven to be stable and to converge to solutions very close to global optima. The time to achieve convergence was consistently less than the time constant of the network's processing elements (neurons). This implies that fast solutions can be realized if the algorithm is implemented in hardware circuits. Three series of battle scenarios are analyzed and discussed in this report. Input data and results are presented in detail. The main advantage of this algorithm is that it can be adapted to either a special-purpose hardware circuit or a general-purpose concurrent machine to yield fast and accurate solutions to difficult decision problems.

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1. INTRODUCTION

The process of effectively utilizing resources during a military engagement is known as battle management/command, control, and communications (BM/C^3). One of the key tasks of BM/C^3 is the optimal assignment of weapons to destroy targets. The effective engagement of incoming offensive intercontinental ballistic missiles (ICBM's) is one of the Strategic Defense Initiative (SDI) BM/C^3 system's main concerns.

The overall system architecture of the SDI defense system is envisioned as one of several defensive layers corresponding to the different phases that occur in the trajectory of the attacking ICBMs. Those are the boost phase, the mid-course stage phase, and the reentry or terminal phase. Within each defensive layer, computers will use information gathered from sensors to detect, classify, and track potential targets. Using this information, the weapons will have to be assigned to destroy the offensive ICBM's before they detonate. The overall goal of the SDI BM/C^3 system is to minimize the expected leakage of warheads penetrating through each defensive layer. The number of weapon platforms, especially during the boost phase engagement, is usually very small in comparison with the number of attacking ICBMs. Thus, the BM/C^3 system faces a resource optimization problem that must be solved accurately and rapidly.

This paper presents a neural network-based algorithm for the optimization of the weapon-target assignment problem (WTAP). In this problem we deal with the so called one-sided many weapons on many targets battle where the defense must select a weapon assignment policy against a completely known offense attack. Although the real strategic defense problem is dynamic, we have focused in this paper on an associated static formulation. This problem is NP-complete.[1]

The static WTAP is defined as follows. There are M weapon platforms and $K(> M)$ targets. Each weapon platform m has a number B_m of interceptors onboard. Each of the B_m interceptors is assumed to possess the same interception performance. Each target k is restricted by the BM/C^3 system to be fired at by at least r_k and no more than R_k interceptors. The probability that any single interceptor launched from weapon platform m destroys target k is denoted by $P_{km} \in [0,1]$ -the single-shot kill probability (SSKP). This probability depends on the characteristics of the interceptor and the target, e.g., maneuverability, guidance performance, target's shield, type of interceptor warhead, etc., and on the relative geometry between the weapon platform and the target. The expected leakage of target k being fired at by an interceptor from a weapon platform m is denoted by $q_{km} = 1 - P_{km}$. It is assumed that once the targets are detected and classified and their locations and velocity vectors are determined, a kill probability matrix P_{km} can be generated rapidly and made available to the local BM/C^3. The objective is

to find an optimal firing assignment matrix, for all weapon platforms to all targets, that minimizes the total expected leakage of targets subject to the constraints of availability and demand of interceptors.

Situations similar to this static WTAP frequently occur in other areas such as operations research, and logistics management. These are generally known in the literature as transportation problems (TP).[2,5] The TP is defined as follows: Given M sources and K destinations with source m capable of supplying amount $\underline{B}_m$ and destination k having demand d_k $(r_k \leq d_k \leq R_k)$, find the least cost transportation pattern from all the sources to all the destinations, when the cost of transporting a unit amount from m to k is q_{km}.

Many efficient algorithms have been described in the literature to solve this problem, e.g., network solution techniques, stepping-stone techniques, alternating basis methods and primal simplex algorithms,[7–13] graph theory-based algorithms,[6,14] and the Hungarian algorithm.[15,16] An efficient algorithm for linear programming that can be applied directly to the standard transportation problem with equality constraints has been developed recently.[17] This algorithm uses a primal-dual state variables approach and a sigmoid function which serves as a "barrier" function for the calculation of its primal variables. When the state variables are integers, the TP is usually referred to as the assignment problem (AP).[2,5,9]

In contrast to the common TP and AP, the WTAP investigated here is a nonlinear combinatorial optimization problem. Its objective function consists of the product of expected leakage values of a target subjected to simultaneous multiple shots from different platforms. A method to solve the nonlinear WTAP has been presented recently.[18]. The underlying idea of this method is to iteratively apply linear approximations to solve the nonlinear problem. The linear approximation used by these authors is applied to the case where each target can receive at most one shot. A maximum marginal return (MMR) algorithm for the WTAP has been presented.[19] The main idea behind this algorithm is to consistently select among all possible weapon-to-target assignments the one which has the greatest decrease in the expected surviving value of each target. The computational complexity of this algorithm has been shown to be $O(K + \ell og(K \cdot \sum_{m=1}^{M} B_m))$. A nonlinear network flow (NNF) algorithm[20] first relaxes the constraint concerning the total number of weapon platforms and solves the resulting convex nonlinear problem via a primal-dual method. After the optimal fractional assignments are determined for each single weapon platform, another algorithm must be utilized to determine near-optimal overall assignments.

Our algorithm is based on the artificial neural networks approach[21,22] combined with the Lagrange differential multipliers method.[30] According to this approach the assignment variables of the problem are defined as the output signals of many interconnected processing elements. Energy functions are defined that describe the

objective function and the constraints in terms of these variables. These functions are added to create the total energy to be minimized. A dynamic system is derived whose attractors are the desired constrained minima of the total energy. The outputs of the neurons at the equilibrium state are the desired optimal assignments. An analog (neural network) circuit can be constructed directly from the dynamic model.

The main advantages of this algorithm are that it can provide answers to complex decision problems directly from a solution of a set of dynamical ODEs; the required programming is simple and straightforward; the algorithm is inherently parallel which allows for implementation on concurrent computation machines; and it can also be adopted to hardware to provide high speed processing.

This paper is organized as follows. In Section 2 we describe the general dynamic WTAP and formulate the static problem that we investigate. The neural network formulation of the static problems is presented in Section 3, and in Section 4 the derivation of the algorithm is described. The question of convergence to equilibria and their stability is briefly addressed in Section 5. In Section 6 we present numerical results for three representative examples. We conclude with a short summary in Section 7.

2. PROBLEM STATEMENT

In order to provide a broader view of the scope of the problem to be solved we first discuss the dynamic WTAP and then focus on the solution of the static problem. We conjecture that good solutions for the dynamic problem can be obtained by combining fast solutions of successive static problems.

The attack we consider consists of different types of ICBMs and/or their warheads which may be launched from various sites. All these targets are located within a finite domain in space referred to as the battle field. All necessary information regarding the type of the targets, their important structural details, and their position and velocity vectors is assumed to be provided to the local BM/C^3 system by the global BM/C^3 system. The local BM/C^3 system then generates the SSKP matrix, $P_{km}(t) \in [0,1]$, for every $k \in \mathcal{K}$ and $m \in \mathcal{M}$. $\mathcal{K}$ is the set of integers which designate each target in the battle field at a given time t; i.e., $\mathcal{K} = \{1, 2, ..., K(t)\}$ and $\mathcal{M} = \{1, 2, ..., M(t)\}$ is the set of integers designating the defense weapon platforms. The local BM/C^3 system also assigns to each target k a value $S_k(t)$ that indicates its relative threat potential. Both $P_{km}(t)$ and $S_k(t)$ are assumed to be updated in real time according to changes in the battle field. The SSKP is considered to be a property of the specified weapon platform-target pair only. A schematic description of the problem is depicted in Fig.1.

Assume that at t=0, K(0) targets are approaching the battle field that is defended by an array of M(0) (M(0) < K(0)) weapon platforms. Each weapon platform m is equipped with $B_m(0)$ interceptors. Let $\hat{Y}(t) = \{\hat{Y}_{km}(t) \in Z^+\}$, $k \in \mathcal{K}, m \in \mathcal{M}$, where $\hat{Y}_{km}(t)$ is an assignment variable that describes the number of interceptors from platform m assigned to be fired at target k at the period of time Δt, between $t' - \Delta t$ and t'. If the solution of the problem is calculated every time step Δt then the overall expected leakage value of targets over time t, is

$$\hat{E}_p(\hat{Y}, t) = \sum_{t'=0}^{t} \sum_{k=1}^{K(t')} S_k(t') \prod_{m=1}^{M(t')} (1 - P_{km}(t'))^{\hat{Y}_{km}(t')} . \tag{1}$$

We assume that each assignment decision results in a statistically independent interception event. No cumulative effects accrue so that, if $\hat{Y}_{km}(t')$ interceptors are assigned to target k, it will survive with a probability $\left(q_{km}(t')\right)^{\hat{Y}_{km}(t')}$ where $q_{km} = 1 - P_{km}$.

Let $L(t') = [L_k(t') \in \{0,1\}]$ denote the set of targets killed and/or targets that escaped the defense during the period of time Δt between $t' - \Delta t$ and t', and let $N(t') = N_\ell(t') \in \{0,1\}$ denote the set of new targets entering the battle field during that time period. Then the set of existing targets at time t is

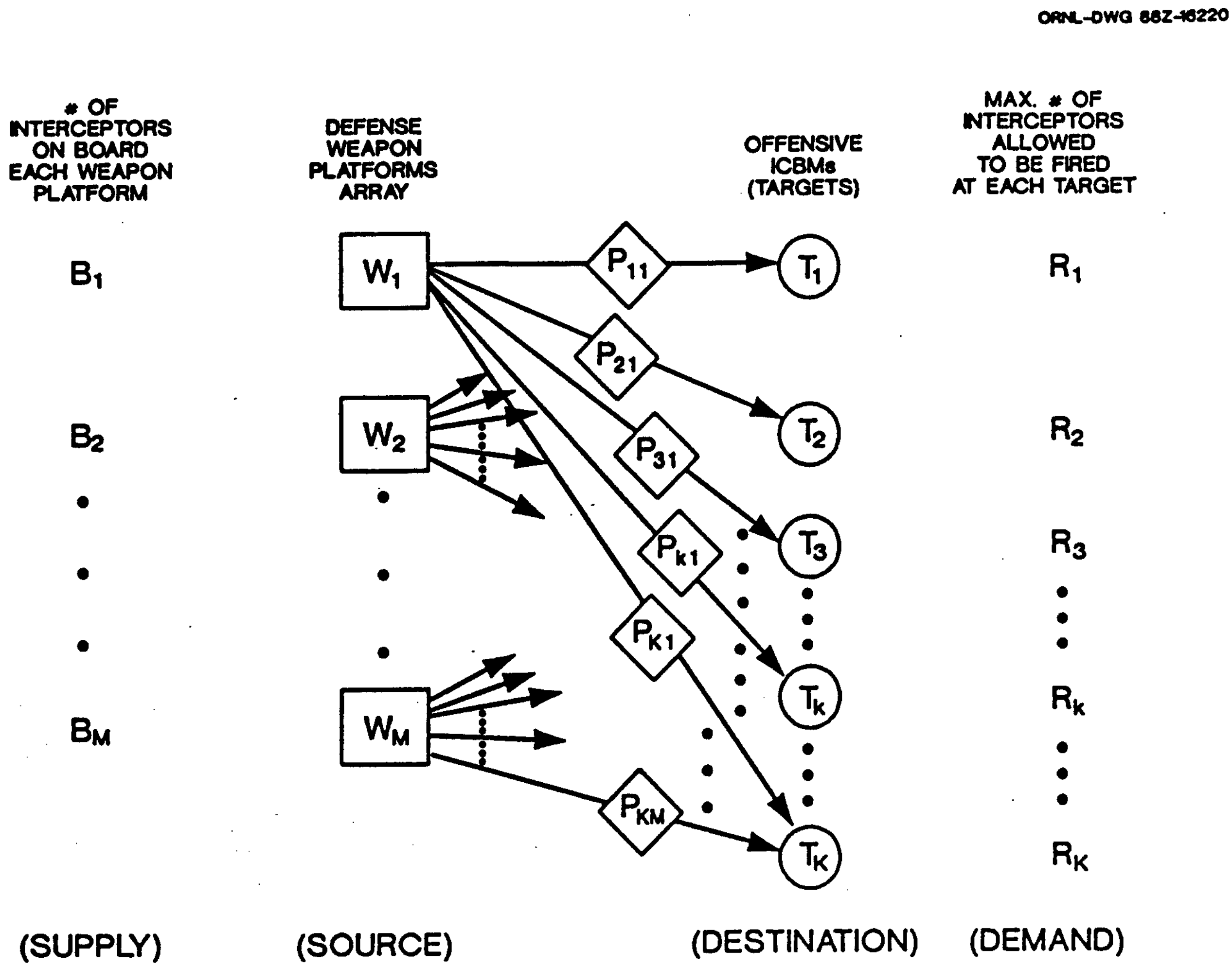

Fig. 1. The Weapon-Target Assignment Problem (WTAP) graph description.

$$K(t) = K(0) + \sum_{t'=0}^{t} \sum_{\ell} N_{\ell}(t') - \sum_{k} L_k(t') \qquad (2)$$

where

$$L_k(t') = min\left(1, \sum_{s=0}^{t'} \sum_{m=1}^{M(t')} \hat{Y}_{km}(s)\right).$$

A target k that has been assigned to be fired at by platform m (i.e., $\hat{Y}_{km}(t) \geq 1$) is considered either destroyed or having escaped the defense, i.e., $L_k(t) = 1$. The number of interceptors onboard each platform m, at time t is

$$B_m(t) = B_m(0) - \sum_{t'=0}^{t} \sum_{k=1}^{K(t')} \hat{Y}_{km}(t') \ . \tag{3}$$

Problem No. 1

The first problem we define is the dynamic WTAP. In this problem the local BM/C[3] must determine the assignment matrix $\hat{Y}(t)$ such that the overall expected leakage value of the offensive targets over the time period $t \in [0,T], \hat{E}_p(\hat{Y},T)$, is minimized, subject to:

(a) $K(t) > 0, \quad t \in [0,T] \quad ;$

(b) $B_m(t) \geq 0, \quad m \in M, \quad t \in [0,T] \quad .$

This is a dynamic many weapon to many target assignment problem. It consists of a nonlinear objective function (1) and inequality constraints (a) and (b). Many uncertainties are involved in the estimation of the ICBMs flight trajectories and those of the interceptors, which cause uncertainty in the values of $P_{km}(t)$ and $S_k(t)$. A closed mathematical solution to the dynamic WTAP appears very difficult, if not impossible.

Problem No. 2

Problem No. 2 is the deterministic assignment problem for a single period of time, Δt, ignoring system dynamics. This leads to a static WTAP, where M weapon platforms are committed to defense against K(>M) targets. Each platform m is equipped with B_m interceptors. We denote the assignment matrix in the static problem by

$$\overline{Y} = \{\overline{Y}_{km} \in Z^+\}, \quad k \in \mathcal{K}, m \in \mathcal{M} \ .$$

Again, the local BM/C[3] system must determine $\overline{Y}$ such that the overall expected leakage value of the offensive targets

$$\bar{E}_p(\overline{Y}) = \sum_{k=1}^{K} S_k \prod_{m=1}^{M} (1 - P_{km})^{\overline{Y}_{km}} \tag{4}$$

is minimized, subject to:

(i) Each target $k \in \mathcal{K}$, is fired at by at least r_k, but no more than R_k interceptors.

(ii) Each weapon platform $m \in \mathcal{M}$, has B_m interceptors.

(iii) The total number of interceptors launched is the maximum possible; i.e., the smaller number of either the total inventory of interceptors, $\sum_{m=1}^{M} B_m$, or the total number of interceptors approved by the BM/C^3 system, $\sum_{k=1}^{K} R_k$.

In trying to solve Problem 2 using a neural network technique, we noticed that a solution for Problem 2 can be constructed from simultaneous solutions of a number of duplicates of a simpler problem (Problem No. 3) defined below. We will discuss this issue in Section 3.

<u>Problem No. 3</u>

This problem is the same as Problem 2 with an additional constraint (iv) restricting each weapon platform to shoot at a given target only one interceptor.

We denote the assignment matrix in this problem by $Y = \{Y_{km} \in (0,1)\}$, $k \in \mathcal{K}, m \in \mathcal{M}$, and determine $\underline{Y}$ such that the overall expected leakage value of offensive targets

$$E_p(Y) = \sum_{k=1}^{K} S_k \prod_{m=1}^{M} (1 - P_{km} Y_{km}) \tag{5}$$

is minimized, subject to: constraints (i–iii) of problem 2 and

(iv) Each weapon platform can shoot at a specified target only one interceptor, but can otherwise shoot at several other targets simultaneously.

<u>Combinatorial Complexity</u>

In order to address the degree of complexity of these problems we estimate the number of possible assignments when all the interceptors are assumed independent and distinct. We also assume that there are K targets and W = Kn + W_o interceptors, and that each target is fired at with at least n and at most n + 1 interceptors. The number of different possible assignments corresponding to this case is equal to the number of ways to fill K boxes with W different objects such that each box will contain either n or n + 1 objects. The number of ways to choose K objects among W such that each box contains only one object is $\frac{W!}{(W-K)!}$. The number of ways to fill each box with the second object is $\frac{(W-K)!}{(W-2K)!}$, with the third

object $\frac{(W-2K)!}{(W-3K)!}$, and so on. The remaining $\underline{W}_o$ objects can be distributed among the $\underline{K}$ boxes in $\frac{K!}{(K-W_o)!}$ ways. Thus, the total number of assignments is

$$\frac{W!}{(W-K)!} \cdot \frac{(W-K)!}{(W-2K)!} \cdot \cdots \cdot \frac{(W-(n-1)K)!}{(W-nK)!} \cdot \frac{K!}{(K-W_o)!} = \frac{W!}{W_o!} \cdot \frac{K!}{(K-W_o)!}$$

In the special case where $W_o = 0$ ($W = nK$) we recover the well known result that the number of possible assignments is W!.

In the WTAP the situation is different since the interceptors are divided into M different groups of B_m identical interceptors. If inequality constraints on the number of shots from each group m are imposed, the total number of possible assignments may exceed that of the previous example.

3. NEURAL NETWORK REPRESENTATION

To address Problem 3 within the framework of artificial neural networks, we adopt the Hopfield-Tank[21,22] approach. We define, instead of the assignment variables Y_{km} of the previous section, new continuous assignment variables

$$X(\bar{t}) = \{X_{km}(\bar{t}) \in (0,1)\}, \quad k \in \mathcal{K}, m \in \mathcal{M}$$

in a state space D_x contained in Euclidean KM-space. Each X_{km} represents the output signal of a neuron-like processing element. Its magnitude expresses the tendency to assign weapon platform m to shoot at target k. The time frame, $\bar{t}$, is related to the neurons' dynamic response. It is not related to the time space of the dynamic physical problem we discussed previously. The states of the processing elements are denoted by $U(\bar{t}) = \{U_{km}(\bar{t}) \in \mathcal{R}\}, \forall k \in \mathcal{K}, m \in \mathcal{M}$ in state space D_u.

From its state through an activation function, the assignment variable is related to the neuron's state as follows:

$$X_{km} = 0.5[1 + tanh(U_{km}/U_{oo}] \tag{6}$$

where U_{oo} is a constant gain coefficient.

Fig. 2. shows a weapon-target assignment matrix in which each row corresponds to a particular target and each column corresponds to a particular weapon platform. The elements of this matrix are the neurons' output signals, $X_{km}(\bar{t})$. A solution of the problem is achieved whenever the state variables converge to an equilibrium state; (U^e, X^e), such that each neuron's output value is either zero or one, i.e.,

$$\lim_{\bar{t}\to\infty} X_{km}(\bar{t}) = \mathrm{X}^e_{km} = \begin{cases} 1 & \text{if weapon } m \text{ is assigned to shoot at target } k \\ 0 & \text{if weapon } m \text{ is \underline{not} assigned to shoot at target } k \end{cases} \tag{7}$$

The set of all x^e is denoted by Ω_x.

As an example, consider the case of 12 targets, and 4 weapon platforms, each equipped with $B_m = 4$ interceptors (Fig. 2). The BM/C^3 system approves at most two shots at each one of the first five targets at each of the rest of the targets. A feasible solution for this problem would appear as a set X^e with values of "ones" and "zeroes" as shown in Fig. 2.

The static WTAP, Problem 3, is formulated in terms of the neural network state variables, as follows:

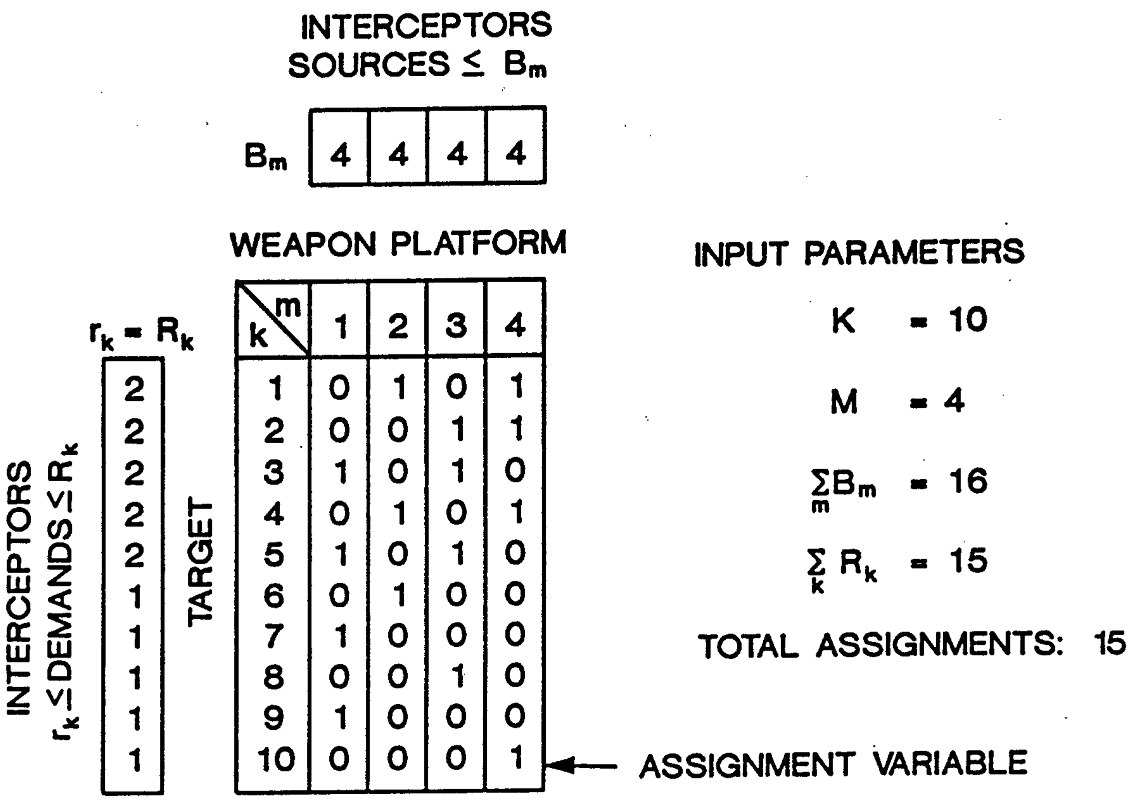

Fig. 2. A weapon-target assignment matrix in the case of (10 × 4) battle scenario.

Minimize

$$E_p(X) = \sum_{k=1}^{K} S_k \prod_{m=1}^{M} (1 - P_{km} X_{km}) \tag{8}$$

subject to:

$$Z_1(k) = \left(\sum_{m=1}^{M} X_{km} \right) - r_k \geq 0\,, \quad \forall \quad k \in \mathcal{K} \tag{9}$$

$$Z_2(k) = \left(\sum_{m=1}^{M} \sum_{\substack{n=1 \\ n \neq m}}^{M} X_{km} X_{kn} \right) - R_k (R_k - 1) \leq 0, \quad \forall \quad k \in \mathcal{K} \tag{10}$$

$$Z_3(m) = \left(\sum_{k=1}^{K} \sum_{\substack{\ell=1 \\ \ell \neq k}}^{K} X_{km} X_{\ell m} \right) - B_m (B_m - 1) \leq 0, \;\; \forall \;\; m \in \mathcal{M} \tag{11}$$

$$Z_4 = \left(\sum_{k=1}^{M} \sum_{m=1}^{M} X_{km} \right) - \psi \leq 0, \psi = min \left(\sum_{k=1}^{K} R_k, \sum_{m=1}^{M} B_m \right) \tag{12}$$

$$Z_5 = \sum_{k=1}^{K} \sum_{m=1}^{M} \mathrm{X}_{km} \left(1 - X_{km}\right) = 0 \tag{13}$$

Formalize (9) and (10) formalize constraint (i), and (11) and (12) constraints (ii) and (iii), respectively. Expression (13) represents an additional constraint which pertains to the specific algorithm we employ. It ensures that the final neuron outputs are 0 or 1. This formulation describes a constrained combinatorial optimization problem mapped onto an artificial neural network.

The extension of the solution of this problem to that of Problem 2 can be obtained if we solve for I_m duplicate target assignment matrices of the form of Fig. 2 simultaneously. I_m is the maximum number of interceptors per platform to be fired at a given target. The inequality constraint related to the interceptors inventory, B_m, can be arbitrarily divided between the duplicate columns of each platform m.

4. ALGORITHM DESCRIPTION

As indicated before, our approach to the solution of Problem 3 [Eqs. (8-13)] is based on a combination of Hopfield and Tank's[21,22] neural network method and the Lagrange multipliers differential method suggested by Platt and Barr.[30] This approach has been exploited previously by us to an NP- complete combinatorial optimization problem with equality constraints (the multiple traveling salesmen problem).[29] According to this method we define energy functions, $E_\alpha(X) : R^n \rightarrow R^+, \alpha = 1, \ldots, 5$ that represent the constraints (9-13). For the inequality constraints (9-12) we introduce, auxiliary functions f2(Z) and $\overline{f2}(Z)$ that are continuously differentiable with respect to $Z_i, i = 1, \ldots, 5$.[22,32] Consequently, the energy functions selected for constraints (9-13) are

$$E_1 = \frac{1}{2} \sum_{k=1}^{K} \overline{f2}[Z_1(k)] \tag{14}$$

$$E_2 = \frac{1}{2} \sum_{k=1}^{K} f2[Z_2(k)] \tag{15}$$

$$E_3 = \frac{1}{2} \sum_{m=1}^{M} f2[Z_3(m)] \tag{16}$$

$$E_4 = \frac{1}{2} f2[Z_4] \tag{17}$$

$$E_5 = \frac{1}{2} Z_5 \tag{18}$$

where

$$f2(Z) = Z^2 H(Z) \quad , \quad \overline{f2}(Z) = Z^2 H(-Z) \tag{19}$$

and H(Z) is the Heaviside function.

The present constrained optimization problem can be converted to an unconstrained problem by introducing Lagrange multipliers, λ_α, and by minimizing the total energy function

$$E_{p\alpha}(X,\lambda) = E_p(X) + \sum_{\alpha=1}^{5} \lambda_\alpha E_\alpha(X) \quad . \tag{20}$$

$E_{p\alpha}$ is a non-convex function, which does not necessarily have extremum points in Ω_x. In order to overcome this difficulty we introduce a modified energy function

$$E(X,\lambda) = E_{p\alpha}(X,\lambda) + E_u(X) \tag{21}$$

where

$$\begin{aligned} E_u(X) &= \sum_{k=1}^{K}\sum_{m=1}^{M} e_u\left(X_{km}\right) = \sum_{k=1}^{K}\sum_{m=1}^{M} \frac{1}{\tau}\int_o^{X_km} U_{km}\left(X_{km}\right) dX_{km} \\ &= \sum_{k=1}^{K}\sum_{m=1}^{M} \frac{U_{oo}}{2\tau}[X_{km}\ell og X_{km} + (1-X_{km})\ell og(1-X_{km})] \end{aligned} \tag{22}$$

and τ is a constant representing the characteristic time response of a neuron. Each (k,m) component, $e_u(X_{km})$ of $E_u(X)$ is a non-positive, continuous and bounded function of X_{km} that vanishes $X_{km} = O$ and $X_{km} - 1$, i.e., $e_u(o) = e_u(1) = 0$. It attains its maximum value at $X_{km} = \frac{1}{2}$; i.e., $e_u\left(\frac{1}{2}\right) = -U_{oo}\ell og\tau$. At the desired constrained minimum state (X^e, λ^e) of $E_{p\alpha}(X,\lambda)$ in Ω_x, the values of $E\left(X^e,\lambda^e\right)$ and $E_{p\alpha}\left(X^e,\lambda^e\right)$ are identical since $E_u\left(X^e\right)$ vanishes.

We now seek a dynamical system of ordinary differential equations (ODEs) which represent the neural network, whose attractors are the desired constrained minima of $E(X,\lambda)$. Since we are dealing with a continuous state problem it seems logical to use the Lagrange multipliers differential approach[29,30] as follows:

$$\begin{aligned} \frac{d\mathrm{X}_{km}}{d\bar{t}} &= -\mu\left(\mathrm{X}_{km}\right)\frac{\partial E}{\partial \mathrm{X}_{km}} \\ &= \mu\left(\mathrm{X}_{km}\right)\left[\frac{\partial E_{p\alpha}}{\partial \mathrm{X}_{km}} + \frac{\partial E_u}{\partial \mathrm{X}_{km}}\right], \;\; \forall \;\; k \in \mathcal{K}, m \in \mathcal{M} \end{aligned} \tag{23a}$$

$$\frac{d\lambda_\alpha}{d\bar{t}} = +\frac{\partial E}{\partial \lambda_\alpha} = E_\alpha, \;\; \forall \;\; \alpha = 1,\ldots,5 \tag{23b}$$

$\mu(X_{km})$ is a modulation function that ensures that the trajectories of (23a) are in D_x. It is selected as the derivative of the sigmoid activation function (6)

$$\mu(X_{km}) = \frac{dX_{km}}{dU_{km}} = [2U_{oo}cosh^2(U_{km}/U_{oo}]^{-1} \tag{24}$$

$\mu(X_{km})$ is a non-negative function that vanishes for $X_{km} = O$ and $X_{km} = 1$; (see Fig. 3).

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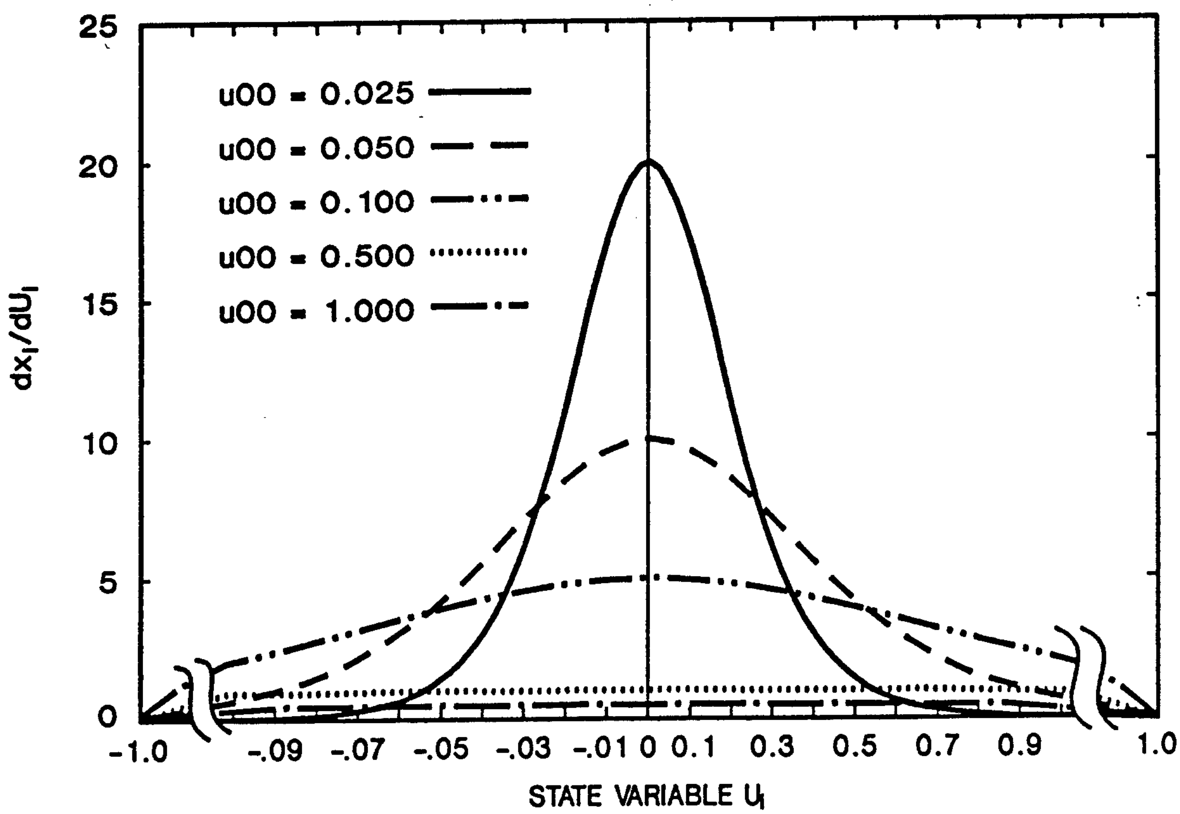

Fig. 3. Variation of the derivative of the sigmoid function versus the neuron state.

For computational simplicity we solve Eq. (23a) for the neuron's state U instead of X. Dividing both sides of Eq. (23a) by $\mu(X)$ we obtain:

$$\frac{dU_{km}}{d\bar{t}} = -[\frac{\partial E_{p\alpha}}{\partial X_{km}} + \frac{\partial E_u}{\partial X_{km}}], \ \forall \ k \in \mathcal{K}, m \in \mathcal{M} \ . \tag{25}$$

Note that $dX/d\bar{t} = \mu(X)dU/d\bar{t}$ and that $\mu(X)$ never really vanishes since $X \in (0,1)$. [21,22]

Performing gradient ascent on λ [Eq. (23b)], has been shown[29,30] to be very effective in the solution of constrained optimization problems. It helps the network's states to "move" towards the surfaces of D_x, while keeping it stable during its entire time evolution.

We have

$$\frac{\partial E_u}{\partial X_{km}} = \frac{\partial e_u}{\partial X_{km}} = \frac{U_{km}}{\tau} \ . \tag{26}$$

This expression adds to the dynamic equation for $U(\bar{t})$ [Eq. (25)] a damping term of the form $-U_{km}/\tau$. This term "pushes" the network states towards a stable equilibria (U^e, λ^e) where all U^e_{km} tend either to $+\infty$ or $-\infty$ and consequently $X^e_{km} \in \Omega_x$, for every $k \in \mathcal{K}, m \in \mathcal{M}$.

Differentiating $E_p, E_1 - E_5$ and E_u with respect to X_{km} and substituting into Eq. (25) we obtain

$$\begin{aligned} \frac{dU_{km}}{d\bar{t}} = -\frac{U_{km}}{\tau} - \{ & -S_k P_{km} \prod_{\substack{j=1 \\ j \neq m}}^{M} (1 - P_{kj} X_{kj}) + \lambda_1 \bar{f}1[Z_1(k)] \\ & + 2\lambda_2 f1[Z_2(k)] \cdot \sum_{\substack{j=1 \\ j \neq m}}^{M} X_{kj} \\ & + 2\lambda_3 f1[Z_3(m)] \cdot \sum_{\substack{\ell=1 \\ \ell \neq k}}^{K} X_{\ell m} + \lambda_4 f1[Z_4] \\ & + 0.5\lambda_5 (1 - 2X_{km}) \}, \forall k \in \mathcal{K}, m \in \mathcal{M} \end{aligned} \tag{27}$$

with

$$f1(Z) \equiv \frac{1}{2}\frac{df2}{dZ} = ZH(Z) \tag{28a}$$

and

$$\overline{f1}(Z) \equiv \frac{1}{2}\frac{d\overline{f2}}{d2} = ZH(-Z) \tag{28b}$$

Equations (23b) and (27) define an autonomous system of ODEs. In general, for any given set of initial conditions, i.e., $U_{km}(0)$ and $\lambda_\alpha(0)$, the system may approach a different limiting pattern (U^e, λ^e) as $t \to \infty$. Since both the objective function, $\mathrm{E}_p(\mathrm{X})$, and the constraints, $\mathrm{E}_\alpha(\mathrm{X})$, are non-convex, the problem may contain many local minima. Therefore, for each given initial state $(U(0), \lambda(0))$, the solution may converge to a different local minimum. In Section 6 we discuss the question of how to choose initial conditions.

5. CONVERGENCE AND STABILITY

Let $X^e \in D_x$ and $\lambda^e \in R^5$ be an equilibrium for Eqs. (27) and (23b). It is called stable if, given any $\epsilon > 0$, there exists $\delta > 0$ such that $\mid X(0) - X^e \mid < \delta$ implies $\mid X(\bar{t}) - X^e \mid < \epsilon$ for all $\bar{t} > 0$, i.e., any trajectory, $X_{km}(\bar{t})$ remains within an arbitrary small distance from X^e_{km}. The equilibrium is called asymptotically stable if there exists $\Delta > 0$ such that $\mid X(0) - X^e \mid < \Delta$ implies that $X(\bar{t}) \to X^e$ as $t \to \infty$, i.e., all trajectories starting close enough to X^e approach X^e asymptotically.

The second (direct) method of Lyapunov[32,33] provides a technique to test the stability of equilibria without solving the system of ODEs. Let $V(x, \lambda) : R^{KM+5} \to R$ be a continuously differentiable function in a neighborhood of (X^e, λ^e), such that

(a) $V(X, \lambda) > 0 \quad if \quad (X, \lambda) = (X^e, \lambda^e)$

(b) $dV/d\bar{t} \leq 0 \quad in \quad R^{KM+5}$, then (X^e, λ^e) is stable.

Furthermore, if also (c) $dV/d\bar{t} < 0$ in R^{KM+5}, then (X^e, λ^e) is asymptotically stable. A function V satisfying (a) and (b) is called a Lyapunov function for the associated dynamical system. It should be mentioned, however, that there is no systematic method of finding Lyapunov functions. In spite of the widespread use of Lyapunov's method, it is still a matter of ingenuity and trial and error to find an appropriate Lyapunov function.

Consider now the scalar function E(X,λ) [Eq.(21)]. All its components are continuous, differentiable, non-negative functions except E_u which is non-positive. We add a complementary positive term to E_u so that we have a non-negative function V(X,λ):

$$\begin{aligned} V(X, \lambda) &= E(X, \lambda) + \sum_{k=1}^{K} \sum_{m=1}^{M} \frac{1}{\tau} \int_{\frac{1}{2}}^{o} U_{km} dX_{km} \\ &= E(X, \lambda) + KMU_{oo} \log 2/2\tau > 0 \end{aligned} \tag{29}$$

Since V(X,λ) satisfies condition (a), it can serve as a candidate Lyapunov function. Its time derivative is:

$$\frac{dV}{d\bar{t}} = \sum_{k=1}^{K} \sum_{m=1}^{M} \frac{\partial E}{\partial X_{km}} \frac{dX_{km}}{dU_{km}} \frac{dU_{km}}{d\bar{t}} + \sum_{\alpha=1}^{5} \frac{\partial E}{\partial \lambda_\alpha} \frac{d\lambda_\alpha}{d\bar{t}} \tag{30}$$

Substitution of $dU_{km}/d\bar{t}$ from Eq. (25) and $d\lambda_\alpha/d\bar{t}$ from Eq. (23b) into Eq. (30) yields

$$\frac{dV}{d\bar{t}} = -\sum_{k=1}^{M}\sum_{m=1}^{M}\frac{dX_{km}}{dU_{km}}\left(\frac{dU_{km}}{d\bar{t}}\right)^2 + \sum_{\alpha=1}^{5}E_\alpha^2 \tag{31}$$

where the derivative dX_{km}/dU_{km} is given by Eq. (24). The first term in the RHS of Eq. (31) is clearly non-positive while the second term is always non-negative. A solution of the problem could be shown to be stable if the absolute value of the first term, $V_1 = -\sum_{k=1}^{K}\sum_{m=1}^{M}\frac{dX_{km}}{dU_{km}}\left(\frac{dU_{km}}{d\bar{t}}\right)^2$ is always larger than the second term $V_2 = \sum_{\alpha=1}^{5}E_\alpha^2$.

Note, that the zeroes of the terms in the RHS of Eq. (31) the stationary points of the network [Eqs. (27) and (23b)].

$dV/d\bar{t}$ was persistently negative in all the numerical experiments and became zero only when the solution converged to equilibrium. Typical behaviors of V_1, V_2 and $dV/d\bar{t}$ are shown in Fig. 7 and Table 4 in the next section. In this example, V_1 is significantly smaller than V_2, and V_2 converges to zero much faster than V_1. This indicates that the network tends to "push" the state variables to fulfill the constraints before their driving forces, $dU_{km}/d\bar{t}$, converge to zero. No trend towards so called "strange attractors" or other kinds of instabilities has been detected during the extensive numerical simulations carried out. The solutions were always stable.

6. RESULTS AND DISCUSSION

In this section we describe the performance of the present algorithm and analyze one representative battle scenario. We consider convergence, stability, optimality and the number n of time iterations required for the solution to converge to a stable equilibrium.

The ratio between τ and the product $n * \Delta\bar{t}$, where $\Delta\bar{t}$ is the simulation time-step, provides a good criterion for adequacy of the present algorithm for real-time processing using special-purpose neurocomputers.

Although many series of tests (battle scenarios) of problem sizes up to 50 x 50 have been carried out, only three examples are described. The tests verify empirically the usefulness of the proposed algorithm and its performance. Several interesting algorithm design issues that arose during these tests were addressed.

Computations were carried out using Euler's first order explicit scheme on a VAX 8600 and PC/XT:

$$U_{km}^{new} = U_{km}^{old} + \Delta\bar{t}\ [dU_{km}/d\bar{t}\ ]^{old} \tag{32a}$$

$$\lambda_{\alpha}^{new} = \lambda_{\alpha}^{old} + \Delta\bar{t}\ [E_{\alpha}]^{old} \tag{32b}$$

The neurons' time constants were chosen equal to one ($\tau = 1$), and all the gain coefficients were chosen equal to 0.01 ($U_{oo} = 0.01$). Initial values for the state variables were chosen, such that all the output signals were zero, $X(0) = 0$. This selection was found to be very effective for our algorithm. It enables the network to evolve rapidly towards local minima and reach good solutions. The time increments, $\Delta\bar{t}$, were taken equal to either 10^{-3} or 10^{-4} s. As the dimensions of the problem increase, smaller values of $\Delta\bar{t}$ are recommended to prevent numerical instabilities. The criterion for convergence to an equilibrium was the absolute relative change of every output signal X_{km} and Lagrange multiplier λ_{α} to be less than 10^{-4}. An additional check of convergence to a feasible solution is carried out automatically when a convergence in time is detected. For the examples tested, convergence to a valid solution was always achieved.

Example 1

Consider a small-scale battle with K = 6 and M = 6. The SSKP matrix is given in Table 1.

Table 1. Input SSKP Matrix (P_{km})

K/M	1	2	3	4	5	6
1	1.00	0.50	0.40	0.60	0.50	0.10
2	0.60	0.40	0.90	0.70	0.30	0.20
3	0.10	0.80	0.30	0.60	0.40	0.60
4	0.50	0.30	0.70	0.20	0.10	0.40
5	0.30	0.20	0.50	0.60	0.80	0.70
6	0.70	0.60	0.40	0.10	0.30	0.20

Three different scenarios of this battle are examined as described in Table 2. In all these instances r_k is kept equal to one. The initial values of the Lagrange multipliers were chosen as follows: $\lambda_1(0) = 10.0, \lambda_2(0) = 1.0, \lambda_3(0) = 5.0, \lambda_4(0) = 10.0, \lambda_5(0) = 0.2$. The results of the optimal total expected leakage value of targets and number of time iterations required to converge to equilibria, are presented in the last two columns of Table 2.

Table 2. Results of three (6 × 6) battle scenarios ($r_k = 1$)

No.	Max # of interceptors approved against each target	# of interceptors onboard each platform	Max # of interceptors approved for this battle	# of interceptors consumed	Fractional expected leakage value	#of time iterations
	R_k	B_m	ψ		Ep/6	n
1.1	1	1	6	6	0.267	80
1.2	1	2	6	6	0.183	82
1.3	2	2	12	11	0.078	245

The final assignment maps of these scenarios are presented in Table 3. In all three cases, the results are optimal. Note, that in case (1.3) the algorithm recommends an assignment of only one interceptor against target 1 in spite of the fact that two are allowed by the BM/C^3 system. This decision is drawn since

Table 3. Final assignment maps for three (6 × 6) battle scenarios ($r_k = 1$)

	Scenario 1.1						Scenario 1.2						Scenario 1.3					
→ m																		
↓	1	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0
K	0	0	0	1	0	0	0	0	1	0	0	0	0	0	1	1	0	0
	0	0	0	0	0	1	0	1	0	0	0	0	0	1	0	1	0	0
	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1	0	0	1
	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1	1
	0	1	0	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0

$P_{11} = 1.0$ and no further reduction in the value of target 1 can be achieved. The algorithm has automatically decided to save one interceptor. The total expected leakage value is reduced in scenario 1.3 by 57% from that of scenario 1.2.

In the case $R_k = 1 = B_m$ for all k and m (scenario 1.1), the present WTAP reduces to the standard linear assignment problem (AP). AP's are commonly formulated in the literature by a linear cost function of the form

$$E_{AP}(X) = \sum_{k=1}^{K} \sum_{m=1}^{M} S_k(1 - P_{km})X_{km} \tag{33}$$

This objective function can be used for the present algorithm. The present objective function given in Eq. (8), however, has been found to be far more effective. It possesses in its gradient term in Eq. (27) an inherent "sorting" mechanism that rapidly can sort out the assignment variables which are associated with the largest P_{km} in each row k for all K rows. This "sorting" mechanism is a fundamental building block of the present optimization algorithm.

The solution of the standard AP which minimizes $E_{AP}(X)$ for $S_k = 1, \forall\ k \in \mathcal{K}$ and a given P_{km} matrix, is also valid for the maximization of:

$$\sum_{k=1}^{K} \sum_{m=1}^{M} P_{km} X_{km} \ .$$

Therefore, it can be used for APs which require, for instance, to maximize a total. Scenario 1.1 is equivalent to the AP described by Tank and Hopfield[35] where the total rate of library work done by a group of six students is to be maximized. The SSKP matrix in Table 1 is equivalent to 1/10 of the matrix of rates of work of this reference. The assignment matrix obtained in Table 3 for this scenario is identical with the solution updated by Tank and Hopfield.[35]

Typical behaviors of the trajectories of states for this example are shown in Figs. 4-6. Fig. 4 shows an oscillation of all the trajectories at the beginning of the evolution of the network for scenario 1.1. It takes the network about 20-25 iterations until the competition between the neurons is decided. Fig. 5 shows the trajectories of four neurons of the third column in the assignment matrix for scenario 1.2 Fig. 6 shows trajectories of four neurons in row 5 in the assignment matrix for scenario 1.3.

The behavior of expressions V_1 , V_2 and $dV/d\bar{t}$ [Eq. (31)] is depicted in Fig. 7. Although this behavior pertains to scenario 1.3, it was found to represent the trend and behavior of these functions in all cases studied. In Table 4, numerical values which describe the time variation of these functions in scenario 1.1 are presented.

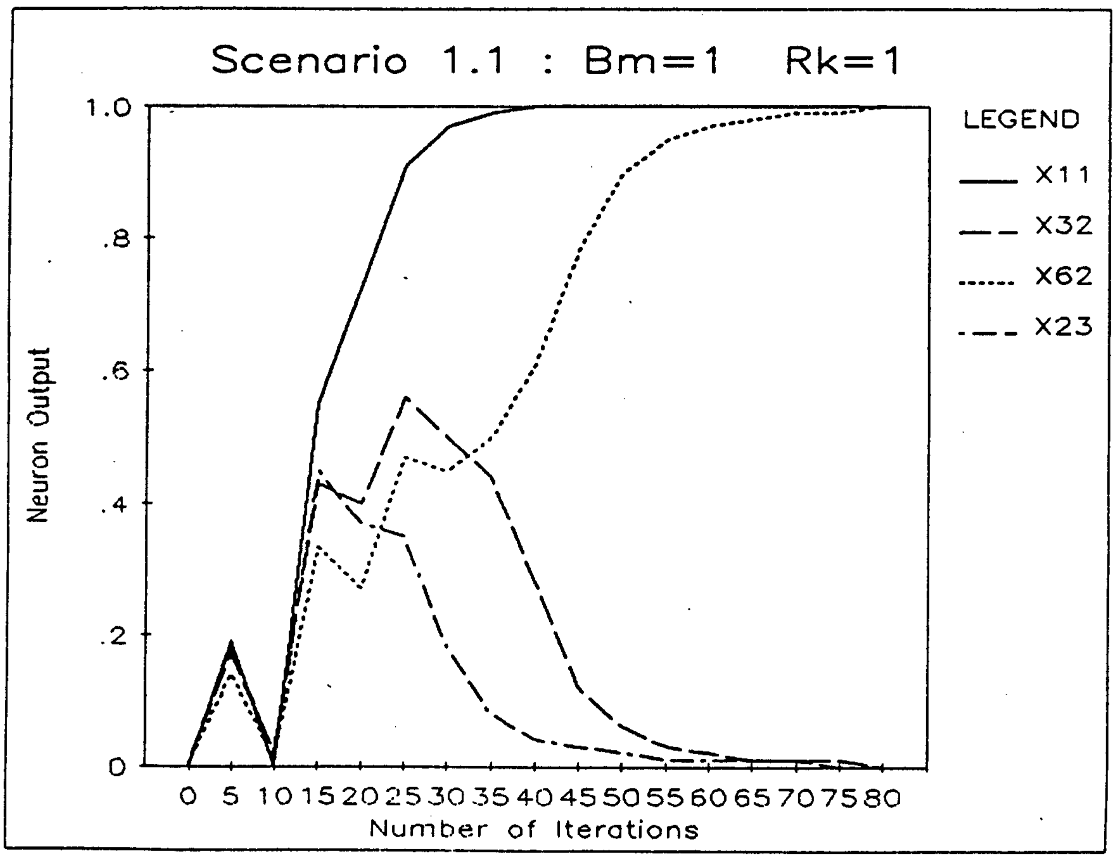

Fig. 4. Trajectories of neuron output signals in a battle scenario 1.1.

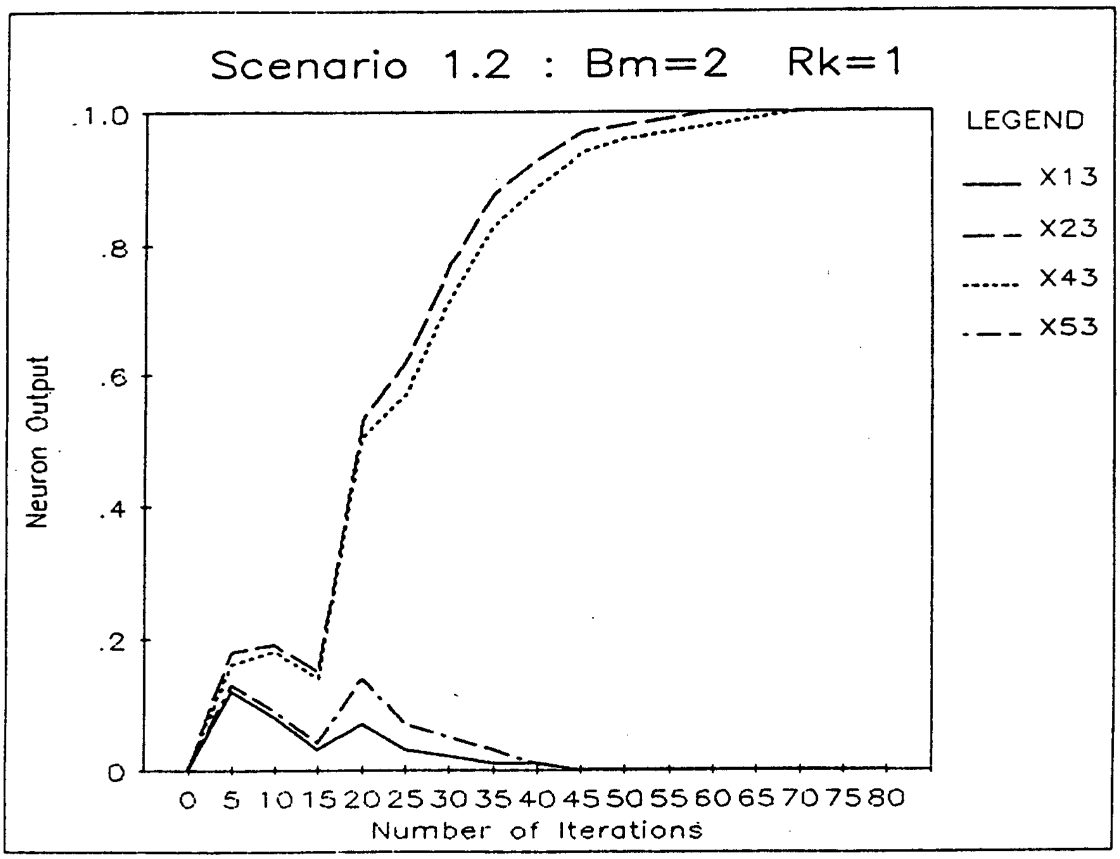

Fig. 5. Trajectories of neuron output signals in a battle scenario 1.2.

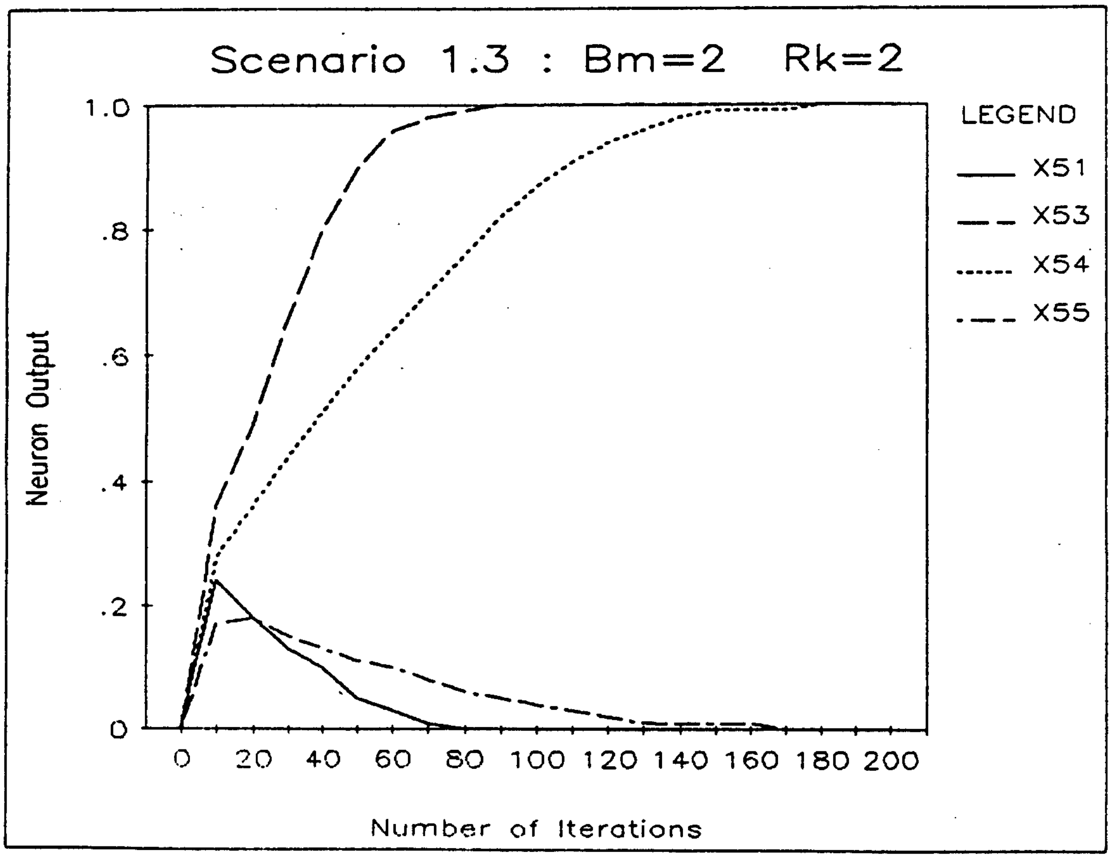

Fig. 6. Trajectories of neuron output signals in a battle scenario 1.3.

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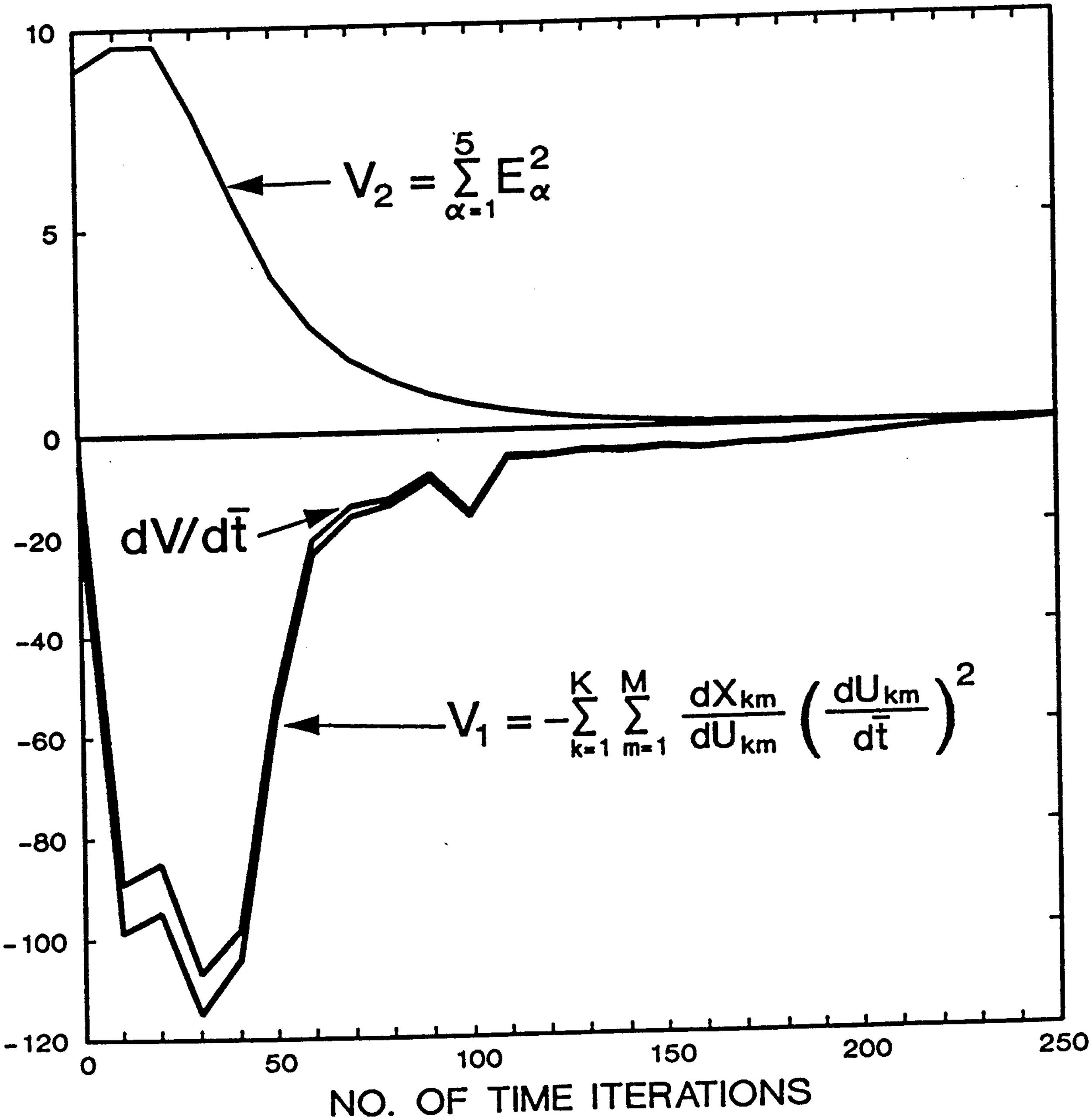

Fig. 7. Variation of V_1, V_2, and $dV/d\bar{t}$ in time in scenario 1.3.

Table 4. Variation of V_1, V_2 and $dV/d\bar{t}$ in time, in scenario 1.1

Iterations	V_1	V_2	$dV/d\bar{t}$
0	−14.806	8.999	−5.807
5	−1436.740	5.074	−1431.666
10	−6357.580	7.239	−6350.341
15	−20225.123	7.290	−20217.832
20	−4577.508	1.641	−4575.867
25	−516.672	1.728	−514.944
30	−161.614	1.038	−160.576
35	−136.408	0.702	−135.707
40	−170.168	0.394	−169.774
45	−111.967	0.134	−111.832
50	−37.427	0.037	−37.390
55	−13.562	0.012	−13.550
60	−5.838	0.005	−5.833
65	−2.984	0.002	−2.982
70	−1.669	0.001	−1.668
75	−0.982	0.001	−0.981
80	0.000	0.000	0.000

Example 2

This example is presented to compare an algorithm with other well-known algorithms. We consider a (10 × 10) AP which is defined as follows: minimize $F = \sum_{k=1}^{10} \sum_{m=1}^{10} q_{km} y_{km}$ subject to $\sum_{k=1}^{10} y_{km} = 1$ and $\sum_{m=1}^{10} y_{km} = 1$, for every $k = 1, \ldots, 10$; $m = 1, \ldots, 10$ and $y_{km} \in \{0, 1\}$. The cost matrix $[q_{km}]$ is given in Table 5.

This problem can be solved with the present algorithm where P_{km} is replaced by $(1 - q_{km})$ and all the constraints' parameters r_k, R_k and B_m are set equal to 1. Using the same initial conditions for $\lambda_\alpha(0)$ as in Example 1, the optimal assignment sequence shown in Table 6 was obtained with the minimum cost of $F = 2.9$. Equilibrium was achieved after 81 time steps. The same minimum cost was obtained with the Hungarian method.[15,16,40] However, different assignments

of equal total costs have resulted from this algorithm for weapons 1, 5, 6, and 9. Somewhat worse results have been obtained from two other methods, i.e., Least cost: $F = 3.1$ and Vogeal's approximation method: $F = 3.3$. Computations with all of these methods were fast due to the relatively small dimensions of the problem.

Table 5. Cost Matrix $[q_{km}]$ for Example 2

k \ m	1	2	3	4	5	6	7	8	9	10
1	.3	.4	.5	.6	.7	.8	.9	.7	.2	.3
2	.2	.4	.4	.4	.5	.6	.7	.9	.7	.8
3	.1	.3	.6	.4	.7	.8	.5	.9	.2	.3
4	.3	.9	.8	.7	.5	.6	.3	.4	.2	.4
5	.7	.8	.9	.9	.6	.5	.4	.7	.3	.5
6	.4	.3	.2	.6	.4	.7	.5	.8	.6	.9
7	.5	.2	.4	.4	.5	.5	.6	.6	.8	.8
8	.3	.7	.4	.6	.5	.8	.7	.9	.7	.6
9	.5	.4	.3	.5	.3	.4	.6	.5	.7	.4
10	.3	.5	.5	.7	.9	.9	.9	.2	.7	.7

Table 6. Assignments Sequences for Example 2

weapon m assigned to $\Rightarrow$	target k (Present algorithm)	target k (Hungarian method)
10	1	1
4	2	2
1	3	8
7	4	4
9	5	3
3	6	6
2	7	7
5	8	9
6	9	5
8	10	10

Example 3

In this example, several defense scenarios are analyzed. An array of $M = 5$ weapon platforms is committed to encounter an attack of $K = 25$ ICBMs. The SSKP matrix provided for this battle is specified in Table 6. Two series of tests were performed (Table 7). For all tests the initial conditions for the Lagrange multipliers were: $\lambda_1(0) = 3.0$, $\lambda_2(0) = 1.0$, $\lambda_3(0) = 0.5$, $\lambda_4(0) = 10.0$, and $\lambda_5(0) = 0.2$.

In the first series, each target was approved to be encountered by only one interceptor ($R_k = 1, k = 1 - 25$), such that a total of 25 interceptors were approved for the whole battle. The results in Table 8 and Figure 8 show a slight reduction in the total expected leakage value from 0.200 to 0.194 when the number of interceptors per platform was increased from 6 to 9, because platform 3 had kill probabilities which were the largest for ten targets out of the twenty-five (see Table 7, column 3). Since nine interceptors were available on this platform, the algorithm successfully assigned them to the appropriate targets.

In the second series, the defense array was allowed to encounter ten designated targets (say, $k = 1 - 10$) with two interceptors, while the other targets ($k = 11 - 25$) were encountered with one interceptor only. Here, a total of ten more (i.e., 35) interceptors were allowed for the battle in the last three scenarios. In the case where each platform was equipped with interceptors, a maximum number of 30 interceptors were available. Therefore, two interceptors were approved for the first five targets only.

Table 7. Input Kill Probability Matrix $[P_{km}]$ for Example 3

0.15	0.21	0.24	$\underline{0.90}$	$\underline{0.37}$
$\underline{0.48}$	0.38	$\underline{0.54}$	0.24	0.09
0.78	0.21	$\underline{0.92}$	0.10	$\underline{0.95}$
$\underline{0.71}$	$\underline{0.65}$	0.00	0.65	0.06
$\underline{0.43}$	0.07	$\underline{0.85}$	0.35	0.12
$\underline{0.62}$	0.24	$\underline{0.49}$	0.36	0.40
0.81	0.10	$\underline{0.94}$	0.82	$\underline{0.91}$
0.23	0.10	0.32	$\underline{0.55}$	$\underline{0.28}$
0.34	0.65	$\underline{0.93}$	$\underline{0.70}$	0.15
0.22	0.26	$\underline{0.90}$	$\underline{0.76}$	0.36
0.53	$\underline{0.99}$	0.03	0.69	0.02
0.65	0.55	0.38	0.87	$\underline{0.90}$
$\underline{0.93}$	0.50	0.68	0.69	0.65
0.33	0.04	$\underline{0.91}$	0.52	0.63
0.08	$\underline{0.81}$	0.40	0.01	0.08
0.18	0.36	0.21	$\underline{0.97}$	0.03
0.61	0.42	$\underline{0.95}$	0.57	0.47
0.59	0.02	0.06	0.51	$\underline{0.75}$
0.56	0.62	0.56	0.30	$\underline{0.88}$
0.03	$\underline{0.79}$	0.35	0.77	0.55
$\underline{0.77}$	0.04	0.10	0.75	0.54
0.29	0.35	0.79	0.67	$\underline{0.71}$
0.18	0.74	0.32	$\underline{0.82}$	0.02
0.43	0.30	0.41	$\underline{0.57}$	0.20
0.12	$\underline{0.35}$	0.44	0.27	0.09

Adding ten or five more interceptors has resulted in a reduction in the total expected leakage value of targets by 9.5%–18.5%, as shown in Table 8. To provide some idea of the final assignments, the assignments of the last scenario of this series ($B_m = 9$) are marked by underlines which are superimposed to their associated P_{km}'s in the SSKP matrix (Table 7). Examining the final assignments shows that the second shots were correctly selected for targets 1–10 such that both assignments in these rows are the largest ones. The detailed results of all other final assignments are not presented here because of lack of space, but can be provided upon request.

Table 8. Results of (25 × 5) Battle Scenarios in Example 3

# of interceptors onboard each platform	$R_k = 1,\ k = 1 - 25$			$^{(*)}R_k = 2, k = 1 - 10; R_k = 1, k = 11 - 25$			reduction in expected leakage value
	Fractional expected leakage value	no. of time iterations	max. no. of interceptors approved for the battle	Fractional expected leakage value	no. of time iterations	max. no. of interceptors approved for the battle	
B_m	$E_p/25$	n	ψ	$E_p/25$	n	ψ	%
6	0.200	194	25	0.181	251	30	9.5
7	0.197	139	25	0.174	467	35	11.7
8	0.195	148	25	0.168	354	35	13.8
9	0.194	113	25	0.158	337	35	18.5

(*) In the case of $B_m = 6$, a total number of 30 interceptors are available for the battle. Therefore, $R_k = 2$ for targets $k = 1 - 5$ only.

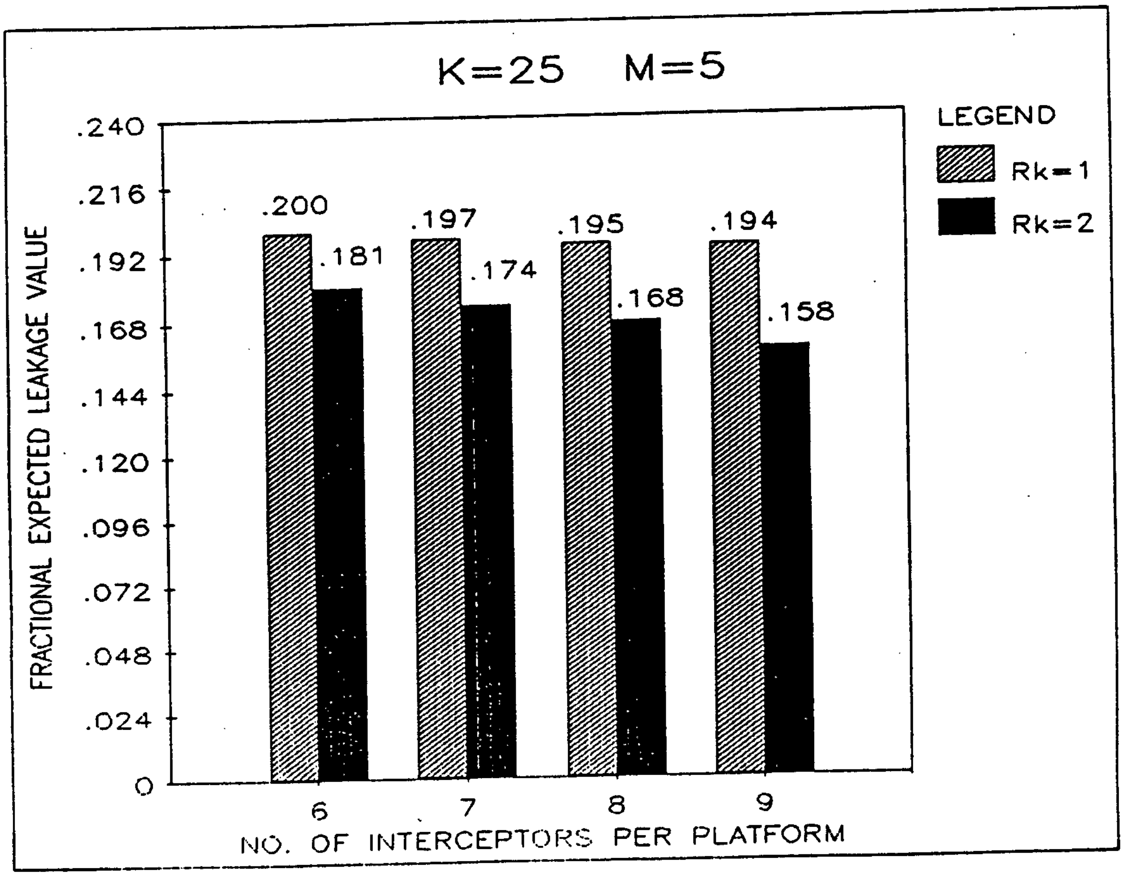

Fig. 8. Variation of the total expected leakage value of targets versus the number of interceptors per platform.

7. SUMMARY AND CONCLUSIONS

A neural network-based algorithm has developed for the solution of the static WTAP. The problem was defined as a minimization of the total expected leakage value of the targets surviving the defense. The minimization was subject to inequality constraints that specify the maximum number of interceptors available on each weapon platform and the minimum and maximum numbers of interceptors specified by the BM/C^3 system required to encounter each target.

The optimization algorithm involves the solution of a system of non-linear ODEs describing the dynamics of a neural network that represents the problem. The trajectories of this system are the desired optimal assignment variables of the problem. The algorithm contains an inherent decision scheme that provides fast and accurate results. The initial conditions of the neurons' outputs have chosen equal to zero.

Many simulations with the present algorithm have shown stable behavior and convergence to feasible solutions. In all cases tested, the solutions were close to global optima. The standard assignment problem can be solved as a particular case of the present problem with $r_k = R_k = B_m = 1$ for every k and m. Convergence times were always less than the time constant ($\tau = 1$) of the network's processing elements.

The results presented in Section 6 show that the algorithm can be useful, not only for the solution of ad-hoc battle situations as required by the BM/C^3 system, but also for pre-war planning and design studies. The effects of parameters such as the number of interceptors per platform, the number of shots at each target and the number of weapon platforms in the array, on the whole defense efficiency can be analyzed. These results can then be used to design an optimal defense array. In these kind of calculations, the computation speed is not a crucial requirement.

The main advantage of this algorithm is that it can be implemented in fast analog circuits. It is estimated that with an implementation in hardware accurate solutions to large-scale WTAPs could be obtained in fractions of a second.

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